Chapter 21 – Appendix

21.1 Hyperbolic Functions

The hyperbolic functions are defined (for x in radians) as follows –

$$\sinh x = \frac{1}{2}(e^x - e^{-x})$$

$$\cosh x = \frac{1}{2}(e^x + e^{-x})$$

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{e^{2x} - 1}{e^{2x} + 1}$$

(Note, for x > 3, $\sinh x \approx \cosh x \approx \frac{e^x}{2}$)

These can be obtained by calculating the required e^x, e^{-x}, etc. with the LL scales and substituting into the above formulae. If x is given in degrees, it must first be coverted to radians.

Exercise 21(a)

(i) sinh 1.7 =

(ii) cosh 54° =

(iii) tanh 30° =

(iv) tanh 1.35 =

21.2 Cursor Lines

These vary for different Slide Rules. Most Slide Rules have marks for converting kW to HP. This is done by setting the kW line over the given value and reading the corresponding HP under its line, or vise versa. The decimal point is lovated by remembering 0.746 kW = 1HP.

Many Slide Rules have a line labelled ‘d’ over the C and D scales and a secong labelled ‘S’ over the A and B scales. (or ‘d’ could be over W scales and ‘S’ over C and D scales). This allows us to set the ‘d’ line over the value for the diameter of a circle and under the ‘S’ line we read off directly the area of the cirlce.
For any other marks and lines, see the instruction leaflet with your Slide Rule.

21.3 Reciprocals

The C and D scales or the W scales can be used by themselves to obtain reciprocals.
Example 1: ¼ = 0.25

1. Place the left index of the C sclae over 4 on the D scale.
2. Above the right index of the D sclae read off 0.25 on the C scale as the answer.

Example 2: ½ = 0.5

1. Place the left (black) index of the W'_1 scale over 2 on the W_1 scale.
2. Below the right (black) index of the W_2 scale read off 0.5 on the W'_2 scale.

Note:

(a) The latter example using the W scales affords the most accurate method for obtaining reciprocals. Combing this with the LL scales method to obtain the decimal point (as outlined in 19.1), we have the ideal way of finding a reciprocal with the Slide Rule.

(b) Sometimes in dividing quantities (e.g. a÷b) it is more convient to do the division upside down (e.g. b÷a), the latter being read off the D scale, usually under the index of the C scale. Thus to obtain the reciprocal, which is the desired value we read it off the C scale above the index of the D scale. A typical example would be if we required $\dfrac{a}{\sin\theta}$ or $\dfrac{a}{\tan\theta}$ when solving a right triangle. It would be much eiser to find $\dfrac{\sin\theta}{a}$ or $\dfrac{\tan\theta}{a}$ if the trignometrical scales are on the body of the Slide Rule.

Example: $\dfrac{8.75}{\tan 35°}$ = 12.5

1. Set the hair line over 35° on the T^1 scale.
2. Place the 8.75 of the C scale under the hair line.
3. Above the left index of the D scale read off 12.5 on the C scale as the value for $\dfrac{8.75}{\tan 35°}$

Exercise 21(b)

(i) $\dfrac{3.6}{\sin 21°}$

(ii) $\dfrac{19.4}{\tan 58.5°}$

(iii) $\dfrac{26.3}{\cos 72°}$

(iv) $\dfrac{23.6}{\sqrt[3]{335}}$

21.4 Addition and Subtraction

A. $a \pm b$.

This can be calculated by noting the relationship $a \pm b = b\left(\dfrac{a}{b} \pm 1\right)$ and using the following procedures.

1. Set the hair line over ‘a’ on the D scale.
2. Place the ‘b’ on the C scale under the hair line.
3. Below the index of the C scale we have the value for $\dfrac{'a'}{b}$ on the D scale.
4. Reset the hair line over $\left(\dfrac{a}{b} \pm 1\right)$ on the D scale.
5. Place the ‘b’ fo the CI scale under the hair line.
6. Below the index of C scale read off $a \pm b = b\left(\dfrac{a}{b} \pm 1\right)$ on the D scale as the answer.

B. $a^2 \pm b^2$

This can be calculated by noting the relationship $a^2 \pm b^2 = b^2\left(\dfrac{a^2}{b^2} \pm 1\right)$ and using the following procedure.

1. Set the hair line over ‘a’ on the D scale.
2. Place the ‘b’ of the C scale under the hair line.
3. Above the index of the B sclae we have the value for $\dfrac{a^2}{b^2}$ on the A scale.
4. Reset the hair line over $\left(\dfrac{a^2}{b^2} \pm 1\right)$ on the A scale.
5. Place the ‘b’ of the CI scale under the hair line.

6. Above the index of the B scale read off $a^2 \pm b^2 = b^2\left(\frac{a^2}{b^2} \pm 1\right)$ on the A scale as the answer.

(Note, this method can be used to solve a right angle triangle using the Theorem of Pathagoras.)

This can be calculated by noting the realationship $\sqrt{a} \pm \sqrt{b} = \sqrt{b}\left(\frac{\sqrt{a}}{\sqrt{b}} \pm 1\right)$ and using the following procedure.

1. Set the hair line over 'a' on the A scale.
2. Place the 'b' of the B scale under the hair line.
3. Below the index of C scale we have the value $\frac{\sqrt{a}}{\sqrt{b}}$ on the D scale.
4. Reset the hair line over $\left(\frac{\sqrt{a}}{\sqrt{b}} \pm 1\right)$ on the D scale.
5. Place the index of the B scale under the hair line.
6. Reset the hair line over 'b' on the B scale.
7. Under the hair line read off $\sqrt{a} \pm \sqrt{b} = \sqrt{b}\left(\frac{\sqrt{a}}{\sqrt{b}} \pm 1\right)$ and the D scale as the answer.

Exercise 21(c)

(i) $8.1^2 - 6.3^2 =$

(ii) $\sqrt{29.1} - \sqrt{19.4} =$

(iii) $45^2 - 31^2 =$

(iv) $\sqrt{65} - \sqrt{49} =$

21.5 Soltuion of Quadradic Equations

For quadradic equation of the form $ax^2 + bx + c = 0$ we recall that if the roots (or solutions) are p and q then:

$$p + q = -\frac{b}{a}$$

and $$pq = \frac{c}{a}$$

Thus to solve a quadradic equation, it amounts to finding two values whose sum and products are known. We do this by successive trials using a rather novel apporach.

(Note, that if we place the right index of the C scale over, for example 9, on the D scale, the for any setting of the hair line the product of the numbers read off under it on the D and CI scale will always equal 9.)

Example: Solve $x^2 - 6x + 8 = 0$

(Note the product of the roots is 8.)

1. Set the right index of the CI scale over 8 on the D scale.
2. (By trial and error, set the hair line in variouis positions, checking each time whether the sum of the values read off the CI and D scale is 6). The correct setting is shown in Fig 20.3 with the solutions x = 2 and 4.

Note:

(a) It is advisable to tabulate the successive trials.

(b) If the quadradic equation has roots of opposite sings (i.e. c < 0) then the roots we are looking for will have a difference of $-\frac{b}{a}$.

Exercise 21(d)
Solving the following for x:

(i) $x^2 + x - 6 = 0$
(ii) $2x^2 + x - 6 = 0$
(iii) $20x^2 – 29x + 6 = 0$
(iv) $2x^2 + 9x - 5 = 0$

21.6 Sine Rule
A. Two Sides and an Angle not Opposite of one of the sides.
Example: Given a = 5.45 cm, b = 4.88 cm and C = 48°.
To find side C and angels A and B.
We have

$$\frac{\sin A}{5.45} = \frac{\sin B}{4.88} = \frac{\sin 48}{c}$$

1. Set the hair line over 48° on the S scale.
2. By trial and error, set an estimate of the value for 'c' on the C scale under the hair line.
   (If we try c = 4.5 on the C scale, the values of A and B on the S scale are read off oppisite the values a = 5.45 and b = 4.88 respectively on the C scale – i.e. A = 64° and B = 53.7°. We then check A + B + C = 64° + 53.7° + 48° = 165.7°. This is short of 180, so we try another value (say c = 4.3) and repeat, checking the sum of the angles again).
   The correct answer will be found to be, side c = 4.24 and Angle A = 73° and B = 59°.

B. Three Sides.
Example: Given a = 48cm, b = 56.6cm and c = 44.2cm.
To find angles A, B, and C.

$$\frac{\sin A}{48} = \frac{\sin B}{56.6} = \frac{\sin C}{44.2}$$

1. We estimate the value for A (say 60°) and set the hair line over this value on the S scale.
2. Place 48 (the value for 'a') under the hair line, and then find the angles B and C corresponding to b = 56.6 and c = 44.2. Checking A + B + C and if it is not 180° try another value for A and repeat steps 1 and 2 again. The correct answers will be found to be A = 55°, B = 76° and C = 49°.

Exercise 21(e)
Using the Sine Rule to find the remaining sides and angles given:

(i) a = 65 cm, b = 51 cm and c = 45 cm.
(ii) a = 60.3 cm, b = 64 cm and c = 58 cm.
(iii) a = 4.15 cm, b = 4.81 cm and C = 71.6°.
(iv) a = 62.5 cm, b = 98 cm and C = 65.2°.